HECKE EIGENVALUES OF SOME GENUS 2 SIEGEL CUSPFORMS

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Let $\mathrm{S}_{j,k}(\mathrm{Sp}_4(\mathbb{Z}))$ be the space of vector valued Siegel modular cusp forms in genus 2 and with coefficients in $\mathrm{Sym}^j \otimes \det^k$ (we follow the convention in [F-VdG]). By a result of Tsushima, for

$$(j,k) \in \{(6,8),(4,10),(8,8),(12,6)\}$$

we have $\dim(\mathrm{S}_{j,k}) = 1$, thus $\mathrm{S}_{j,k}$ is generated by an eigenform $\Delta_{j,k}$. For each prime $p \leq 31$, we give below the eigenvalue of the standard Hecke operator[1] T_p on $\Delta_{j,k}$, obtained (by very different methods!) in [F-VdG] and [C-L].

p	$\Delta_{6,8}$	$\Delta_{8,8}$	$\Delta_{12,6}$	$\Delta_{4,10}$
2	0	1344	-240	-1680
3	-27000	-6408	68040	55080
5	2843100	-30774900	14765100	-7338900
7	-107822000	451366384	-334972400	609422800
11	3760397784	13030789224	3580209624	25358200824
13	9952079500	-328006712228	91151149180	-263384451140
17	243132070500	5520456217764	-11025016477020	-2146704955740
19	595569231400	-28220918878760	-22060913325080	43021727413960
23	-6848349930000	79689608755152	195863810691120	-233610984201360
29	53451678149100	-1105748270340	-1743496339579620	-545371828324260
31	234734887975744	1851264166857664	1979302106496064	830680103136064

References

[C-L] G. Chenevier & J. Lannes, *Kneser neighbors and orthogonal galois representations in dimensions* 16 *and* 24, Oberwolfach Report (2011), article plus complet en cours de rédaction.

[F-VdG] C. Faber & G. V. d. Geer, *Sur la cohomologie des systèmes locaux sur les espaces de modules des courbes de genre 2 et des surfaces abliennes II*, C. R. Math. Acad. Sci. Paris 338 (2004), no. 6, 467–470.

[1]Let ℓ be a prime and let $\rho_{\Delta_{j,k},\ell} : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GSp}_4(\mathbb{Z}_\ell)$ be the ℓ-adic Galois representations associated to $\Delta_{j,k}$ by Weissauer, normalized so that the Hodge-Tate numbers are $0, k-2, j+k-1, j+2k-3$ at ℓ. Then $\rho_{\Delta_{j,k},\ell}$ is unramified outside ℓ and for a prime $p \neq \ell$, the trace of an arithmetic Frobenius Frob_p in $\rho_{\Delta_{j,k},\ell}$ coincides with the eigenvalue of T_p on $\Delta_{j,k}$.